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R. K. Kapania
Associate Editor

Free-Vibration Analysis of Turbine Blades Using Nonlinear Finite Element Method

D. Dhar* and A. M. Sharan†
Memorial University of Newfoundland, St. John's, Newfoundland A1B 3X5, Canada

Nomenclature

c	= specific heat
$[CP]$	= capacitance matrix
e	= element
$\{F_c\}$	= force vector due to convection
$\{F_r\}$	= force vector due to radiation
G	= global
h	= convective heat transfer coefficient
$[K_s]$	= conventional stiffness matrix
$[K_\sigma]$	= stress stiffness matrix
$[KC]$	= conduction matrix
k_x, k_y, k_z	= thermal conductivities in the x, y, and z directions, respectively
l_x, l_y, l_z	= direction cosines normal to the surface
$[M]$	= mass matrix
Q	= heat generated within the body
q	= specified heat flux
$\{T\}$	= nodal temperature vector
T_∞	= gas temperature
t	= time
ϵ	= emissivity of the body
λ	= eigenvalue of the system
σ	= Stefan–Boltzmann constant

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*Graduate Student, Faculty of Engineering and Applied Sciences.
†Professor, Faculty of Engineering and Applied Sciences.

I. Introduction

BEFORE designing a turbine blade, it is essential to know its undamped natural frequencies, and there are various reasons for it. Right from the start until it comes to a constant operating speed, the rotor goes through the various blade natural frequencies. To avoid resonance, the steady-state operating speed should not match the system natural frequencies. As the temperature of the turbine blade changes with time, there is a change in the material properties of the turbine blade. This change in the material property of the turbine blade is reflected in the change in the conventional stiffness matrix $[K_s]$ of the blade, and this causes a change in the natural frequencies of the turbine blade. There is also a significant change in the natural frequencies because of the variational effect of the stress stiffness matrix $[K_\sigma]$, which is nonlinear in nature and is caused by higher deflections during the rotation of the turbine blade.

Sato[1] used the Ritz method to study the effect of axial force on the frequencies of blades with ends restrained elastically against rotation. Sisto and Chang[2] formulated a finite element model to calculate the natural frequencies of the blade. Their model, however, was appropriate for thin and high-aspect-ratio blades only. Sharan and Bahree[3] did the transient-free-vibration analysis of the turbine blade using 20-noded finite elements, which included the effect of the two-dimensional change in temperature. The effect of the stress stiffness matrix, caused by the rotation of the turbine blade, was not included in their study. Warikoo[4] analyzed the propeller-shaft transverse vibrations. His work included the nonlinear stiffness variation but did not include the effect of change in the temperature on the transient natural frequencies of the turbine blade. Dhar[5] carried out extensive studies of the three-dimensional heat transfer process in the turbine blades. He found that there was significant temperature gradient along the height of the blade that was not accounted for in the earlier two-dimensional studies.

In the present investigation, the authors study the combined effects of three types of nonlinearities (the radiative heat flux, the nonlinear stiffness matix, and the material property variation due to the change in the temperature) on the natural frequencies of the turbine blade. Instead of Sharan and Bahree's two-dimensional heat transfer model,[3] a three-dimensional heat transfer model has been used in the present work.

II. Mathematical Formulation

To obtain the blade frequencies as a function of time, one has to solve the eigenvalue problem, which can be stated as

$$[K^G]\{x\} - \lambda[M^G]\{\ddot{x}\} = 0 \tag{1}$$

where $[K^G]$ is obtained by adding $[K_s^G]$ and $[K_\sigma^G]$. Along with this, one also has to solve for the transient temperatures, which change the elastic properties of the material. The three-dimensional nonlinear heat transfer equation is given by

$$k_x\frac{\partial^2 T}{\partial x^2} + k_y\frac{\partial^2 T}{\partial x^2} + k_z\frac{\partial^2 T}{\partial z^2} + Q = \rho c\frac{\partial T}{\partial t} \tag{2}$$

and its nonlinear boundary condition as

$$k_x\frac{\partial T}{\partial x}l_x + k_y\frac{\partial T}{\partial y}l_y + k_z\frac{\partial T}{\partial z}l_z + h(T - T_\infty) + q + \sigma\epsilon(T^4 - T_\infty^4) = 0 \tag{3}$$

The mathematical details of obtaining the global equations using the finite element analysis for Eqs. (1) and (3) are given elsewhere.[5] The global equation for Eq. (3) can be written as

$$[CP^G]\frac{\partial\{T^G\}}{\partial t} + [KC^G]\{T^G\} = \{F_c^G\} + \{F_r^G\} \tag{4}$$

At first, one solves for the temperatures at a given instant of time, and then the material properties are evaluated to obtain the frequencies by solving for the eigenvalues as stated in Eq. (1).

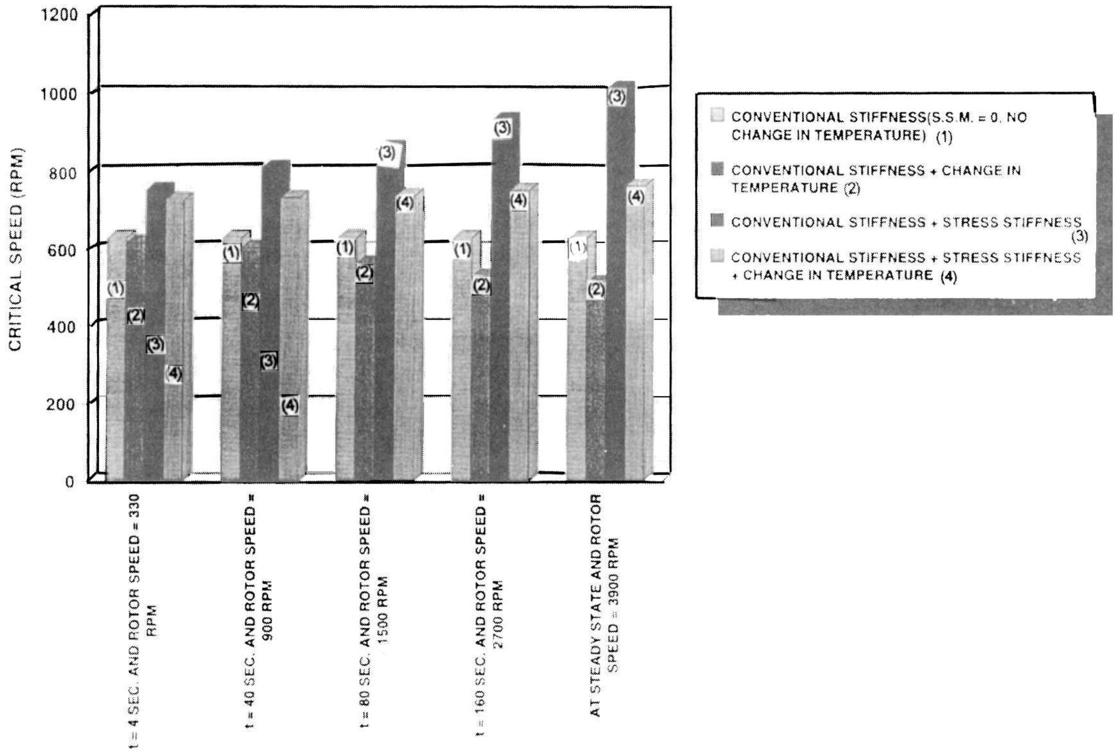

Fig. 1 Comparison of first critical speed obtained from various methods.

III. Results and Discussion

One objective of the present investigation was to show the effect of various nonlinearities (the material property variation with temperature, the nonlinear stiffening of the blade due to larger deflections, the radiative boundary condition) on the natural frequencies of the turbine blade. Besides the effect of the stress stiffness matrix, it was observed that the frequencies of the turbine blade also changed with the change in the temperature. This is due to the change in the elastic properties of the blade. The variation of the material properties with temperature are given elsewhere.[5] Figure 1 shows the variation of the first natural frequencies of the turbine blade at different instants of time during the heating process. Here, the legend shows four types of situations: the number 1 represents frequencies at different instants of time if there were no changes in temperatures, and also, if the stress stiffness matrix was neglected. The number 2 represents the frequencies that are computed with the inclusion of the heat transfer process. The number 3 represents the case when the heat transfer process is neglected but the stress stiffening effect is included. Finally, the number 4 represents the frequencies where all effects are included.

It is very clear that as the material is heated, the frequencies start decreasing because of a decrease in the value of the modulus of elasticity of the material. If the effect of the stress stiffness matrix is not included, then the frequencies would decrease with increase in temperature of the turbine blade. However, the turbine blade rotation causes the numbers within the stress stiffness matrix to increase, which increases the frequencies, as the speed increases. The net effect of all of these factors is that the frequencies increase. Thus, there is a significant change in the natural frequencies in the transient state.

IV. Conclusions

The natural frequencies of the turbine blade were calculated after taking into account the effects of including the stress stiffness matrix to the conventional stiffness matrix and the change in the material properties of the turbine blade with the change in temperature. The nonlinear radiative flux also was included in the analysis. It was observed that the inclusion of these three factors resulted in a significant change in the natural frequencies of the turbine blade in the transient state.

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R. K. Kapania
Associate Editor